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ABSTRACT

The theory of planar near-field measurement is discussed for two different approaches: The scattering-matrix formulation approach and the reciprocity theorem approach. The interaction of test antenna and probe antenna is calculated, in which is taken account of the presence of the probe used to sample the field distributions since the probe is not assumed to be ideal (elementary magnetic or electric dipole), it is said that probe correction is applied.
It is shown that both approaches lead to equal results. The derivation of far-field antenna characteristics from near-field data is then discussed and near-future research items are shortly mentioned.

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SAMENVATTING

De theorie van planaire nabije veld metingen wordt besproken voor twee
benaderingswijzen: De verstrooiings-matrix-formulering benadering en de
reciprociteits-theorema benadering. De interactie van testantenne en
probe is berekend, waarbij rekening is gehouden met de aanwezigheid van
de probe welke gebruikt wordt om monsters van de veldverdeling te meten.
Daar de probe niet ideaal is verondersteld (elementaire magnetische of
elektrische dipool), wordt wel gezegd dat probe-correctie is toegepast.
Aangetoond wordt dat beide benaderingswijzen tot dezelfde resultaten
leiden. Uiteindelijk wordt de afleiding van verre veld antenne-
eigenschappen uit nabije veld data besproken en onderzoeksgebieden voor
de nabije toekomst worden kort genoemd.

CONTENTS

1 INTRODUCTION

Since the 1970s there has been an increasing interest in near-field measurement techniques for predicting far-field antenna properties. This is due to the advantages these techniques offer in comparison with conventional techniques (far-field, compact range)[1, p.xiv; 2, pp.594-596].

In these near-field measurement techniques, the field of the Antenna Under Test (AUT) is detected in phase and amplitude by a probe antenna, scanning a surface which is often be only a few wavelengths away from parts of the antenna structure. The near-field measurement technique perhaps most fully developed and most easy to implement is the planar one and will be discussed in the remainder of this report.

The planar near-field measurement technique, that includes the signal reception in phase and amplitude by the probe, will be described for two approaches.

In the first approach, Kerns [1] uses a scattering-matrix formulation known from microwave circuit theory. The matrices relate amplitudes and phases of waveguide modes and expansion coefficients by linear matrix transformations.

In the second approach, Paris, Leach & Joy [3] use a Lorentz reciprocity theorem formulation. The AUT and the probe are enclosed in proper surfaces so that a source-free volume is created. Then, using the Lorentz reciprocity theorem for a source-free volume, an expression for the received signal is obtained.

After the description of these two approaches, their equivalence will be proven and the derivation of far-field antenna characteristics, using the results of near-field measurements, will be treated.

2   SCATTERING MATRIX FORMULATION APPROACH

For the scattering-matrix formulation approach, the field of the AUT
will be described as an expansion of plane waves. Before this expansion
is given, the coordinate system of AUT and probe must be given, as well
as some wave parameters in these coordinate systems.

2.1   Geometrical configuration

Figure 1 shows the transmission system set-up for the planar near-field
measurement technique [1, p.598]:

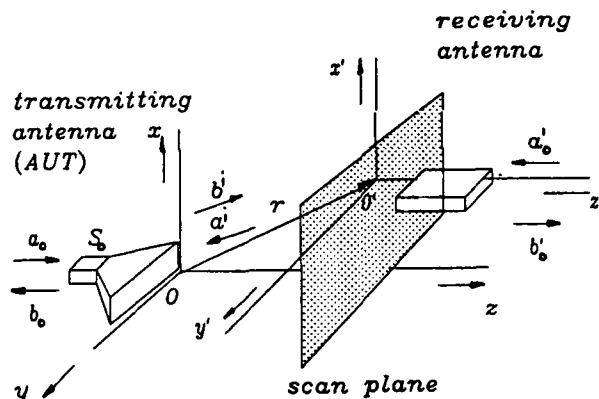Fig.1 Transmission system set-up for planar near-field measurement

Normally the AUT is operated as a transmitting antenna and the probe as
a receiving antenna. The aperture of the probe is moved over a planar
surface, called the scan plane. Samples of the electric field, generated
by the AUT, are taken in equidistant points, that are arranged in a
rectangular grid. The properties of the AUT are described in a

rectangular xyz coordinate system with its origin at O, as shown in
figure 1.

The scan plane is placed parallel to the xy-plane at a distance $z_0 > 0$.
The position of the probe is characterised by the point O', which has
the position vector r_0 with coordinates (x_0, y_0, z_0). The properties of
the probe are described in a rectangular x'y'z' coordinate system with
its origin at O' and its x'-, y'- and z'-axis parallel to the x-, y- and
z-axis respectively. When the probe is moved, its orientation and z_0 are
kept constant while x_0 and y_0 are varied.

The antennas are considered as two-port transducers. For the
transmitting antenna one port is placed in the feed line at S_0 while the
other port is chosen to be at the antenna-aperture. The quantities a_0
and b_0 are phasor wave amplitudes for incident and emergent travelling
waves of a single waveguide mode at S_0. The quantities a^i and b^i are
spectrum density functions for incoming and outgoing waves as defined in
the following. As indicated in figure 1, primes are used to associate
symbols with the probe.

2.2 Expansion in planar waves

The electromagnetic fields in a source-free, linear, homogeneous,
isotropic medium must satisfy the Maxwell equations (an $\exp(-j\omega t)$ time-
dependence is assumed 1))

$$\nabla \times \underline{E} = j\omega\mu\underline{H} \tag{1a}$$

$$\nabla \times \underline{H} = -j\omega\epsilon\underline{E} \tag{1b}$$

$$\nabla \cdot \underline{E} = 0 \tag{1c}$$

$$\nabla \cdot \underline{H} = 0 \tag{1d}$$

---

1) This is the opposite of most electrical engineering formulations, but
is used here because it is consistent with the original derivation of
the plane wave theory [2] and is adopted by Newell [4], upon whose
papers much of the work to come will rely·

so that a general plane wave will have the form:

$$\underline{E} = \underline{A}(\underline{k})\exp(+j\underline{k}\cdot\underline{r}) \qquad (2a)$$

$$\underline{H} = (\omega\mu)^{-1}\underline{k} \times \underline{A}(\underline{k})\exp(+j\underline{k}\cdot\underline{r}) \qquad (2b)$$

with:

$$k^2 = \underline{k}\cdot\underline{k} = k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\epsilon \qquad (3a)$$

$$\underline{k}\cdot\underline{A}(\underline{k}) = k_x A_x + k_y A_y + k_z A_z = 0 \qquad (3b)$$

in which equation (3b) originates from (1c).

The propagation vector $\underline{k}$ is regarded as a function of its transverse components k_x, k_y (which are chosen real). The z-component is thus:

$$k_z = \pm\gamma; \qquad \gamma = \begin{cases} (k^2-k_x^2-k_y^2) & \text{if } k_x^2+k_y^2 \le k^2 \\ -j(k_x^2+k_y^2-k^2) & \text{otherwise} \end{cases} \qquad (4a)$$

The transverse part of $\underline{k}$ will be denoted $\underline{K}$, so that $\underline{K} = k_x\underline{a}_x + k_y\underline{a}_y$ and:

$$\gamma = (k^2 - K^2)^{1/2} \qquad (4b)$$

γ will be taken positive for K<k, negative imaginary for K>k. In (4a), $+\gamma$ is associated with a plane wave travelling into the positive z-direction, $-\gamma$ is associated with a plane wave travelling into the negative z-direction (of course under the condition that γ is real). In virtue of the relation $\underline{k}\cdot\underline{A}(\underline{k}) =0$, (2) yields just two linearly independent fields, hence just two basis fields, for any given $\underline{k}$. The basis vectors $\underline{A}$ (that also indicate the polarization of the $\underline{E}$-field according to equation (2a)) are chosen to be vectors parallel or perpendicular to the plane of $\underline{k}$ and $\underline{a}_z$, which is the plane of incidence for a wave incident on any plane z=constant (see figure 2).
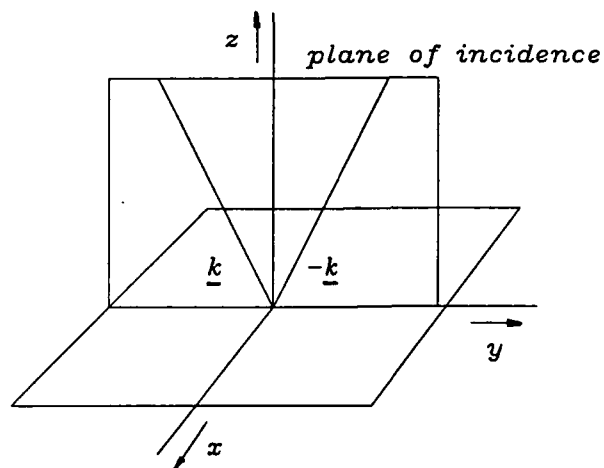
Fig.2 Plane of incidence for a wave incident on plane z=0

The basis fields $\underline{A}$ are set up with the transverse unit vectors:

$$\underline{K}_1 = \underline{K}/K \qquad \text{(5a)}$$
$$\underline{K}_2 = \underline{a}_z \times \underline{K}_1 \qquad \text{(5b)}$$

which are respectively in and perpendicular to the plane of $\underline{k}$ and $\underline{a}_z$. Both vectors $\underline{K}_1$ and $\underline{K}_2$ are in the xy-plane. They are illustrated in figure 3.
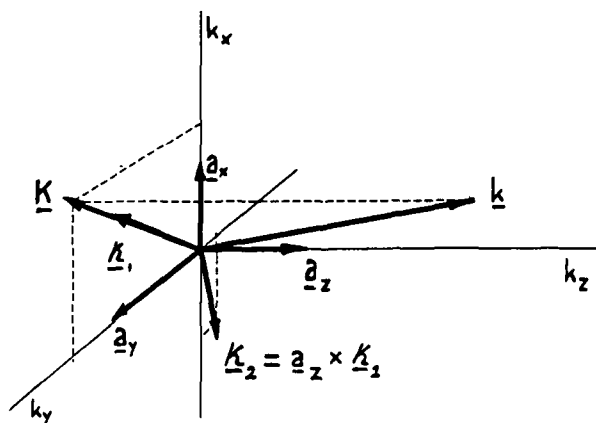
Fig.3 Propagation vector and transverse unit vectors

The basis vectors, that must satisfy $\underline{\mathbf{k}} \cdot \underline{\mathbf{A}} = 0$, are chosen to be: $\underline{\mathbf{A}}_1 = \underline{K}_1 \mp K\gamma^{-1}\underline{\mathbf{a}}_z$, which is parallel to the plane of incidence, and $\underline{\mathbf{A}}_2 = \underline{K}_2$, which is perpendicular to the plane of incidence. Substituting $\underline{\mathbf{A}}_1$ and $\underline{\mathbf{A}}_2$ in (2) gives:

$$\underline{\mathbf{E}}_1^{\pm} = [\underline{K}_1 \mp K\gamma^{-1}\underline{\mathbf{a}}_z]\exp(+j\underline{\mathbf{k}}\cdot\underline{\mathbf{r}}) \tag{6a}$$

$$\underline{\mathbf{H}}_1^{\pm} = \pm\eta_1\underline{\mathbf{a}}_z \times \underline{K}_1\exp(+j\underline{\mathbf{k}}\cdot\underline{\mathbf{r}}); \quad \eta_1 = \omega\epsilon/\gamma \tag{6b}$$

$$\underline{\mathbf{E}}_2^{\pm} = \underline{K}_2\exp(+j\underline{\mathbf{k}}\cdot\underline{\mathbf{r}}) \tag{6c}$$

$$\underline{\mathbf{H}}_2^{\pm} = [\pm\eta_2\underline{\mathbf{a}}_z \times \underline{K}_2 + K(\omega\mu)^{-1}\underline{\mathbf{a}}_z]\exp(+j\underline{\mathbf{k}}\cdot\underline{\mathbf{r}});$$
$$\eta_2 = \gamma/(\omega\mu) \tag{6d}$$

in which the plus-sign indicates a wave travelling into the positive z-direction and the minus-sign indicates a wave travelling into the negative z-direction.Note that ($E_1^{\pm}$, $H_1^{\pm}$) represents a TM-wave and ($E_2^{\pm}$, $H_2^{\pm}$) represents a TE-wave.

The *transmitted* field $E(r)$ of an antenna in the region z>0 for a specified propagation vector k, can now be given as a combination of fields E_1^{+} and E_2^{+}. So the plane wave spectrum expansion is given by:

$$E(r) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} [b^1(k_x,k_y)E_1^{+}(K,r) + b^2(k_x,k_y)E_2^{+}(K,r)]dk_x dk_y$$

(7)

in which the coefficients $b^1(k_x,k_y)$ and $b^2(k_x,k_y)$ are referred to as spectrum-density functions of outgoing waves.

In order to find an expression for the signal $b_0{}'$, received by the probe (see figure 1), it is desirable to express the radiated field $E(r)$ as given in equation (7) in terms of $E_1^{+}(K,r')$ and $E_2^{+}(K,r')$ associated with the x'y'z' coordinate system. To do this, use is made of the relation

$$r = r_0 + r'$$

(8)

that describes the relation between xyz- and x'y'z'-coordinate system. This relation is found by inspection from figure 4.
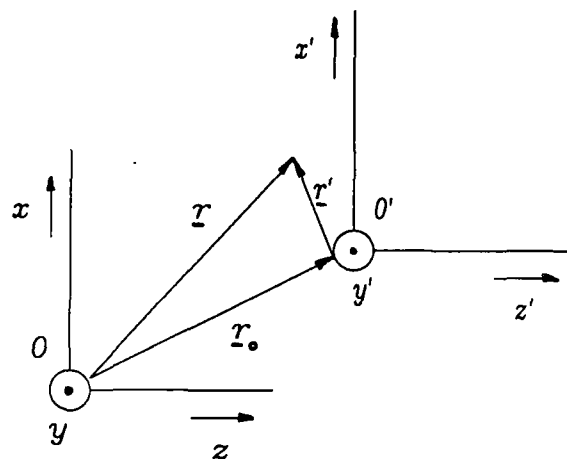
Fig.4 Derivation of translation theorem

With (8) substituted in (7) it is found that:

$$\mathbf{E}_i^+(\mathbf{K},\mathbf{r}) = \mathbf{E}_i^+(\mathbf{K},\mathbf{r}')\exp(+j\mathbf{k}\cdot\mathbf{r}_0); \quad i = 1, 2 \tag{9}$$

and with (9) substituted in (7), the field incident upon the probe is found

$$\mathbf{E}(\mathbf{r}') - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [b^1(k_x,k_y)e^{+j\mathbf{k}\cdot\mathbf{r}_0}\ \mathbf{E}_1^+(\mathbf{K},\mathbf{r}') +$$

$$b^2(k_x,k_y)e^{+j\mathbf{k}\cdot\mathbf{r}_0}\ \mathbf{E}_2^+(\mathbf{K},\mathbf{r}')]dk_x dk_y -$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[a'^1(k_x,k_y)\mathbf{E}_1^+(\mathbf{K},\mathbf{r}') + a'^2(k_x,k_y)\mathbf{E}_2^+(k_x,k_y)]dk_x dk_y$$

$$(10)$$

where

$$a'^i(k_x,k_y) - b^i(k_x,k_y)\exp(+j\mathbf{k}\cdot\mathbf{r}_0); \quad i - 1,\ 2 \qquad (11)$$

are the spectrum density functions of the waves incoming on the probe.

2.3   Coupling equation

A two-port microwave network, as given in figure 5, can be described with a 'scattering-matrix', that relates output wave quantities to input wave quantities of the two-port:

$$b_0 - S_{00}a_0 + S_{01}a_1 \qquad\qquad (12a)$$
$$b_1 - S_{10}a_0 + S_{11}a_1 \qquad\qquad (12b)$$

in which a_0, a_1, b_0 and b_1 are incident and emergent wave amplitudes on the two terminal surfaces, 0 and 1.
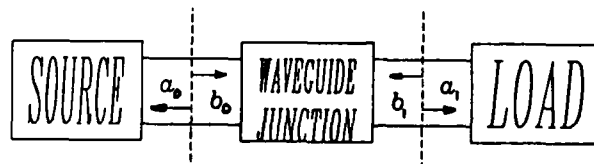
Fig.5 Two-port microwave network

S_{00} and S_{11} are the reflection coefficients of the two-port; S_{01} and S_{10} are the transmission coefficients.

The scattering-matrix formulation is not restricted to two-ports, it can be applied to any N-port microwave-network ($N = 2, 3, \ldots$).

An antenna is viewed as a multiport transducer, having only one input port and *for each polarisation and direction in space* one output port. This transducer transforms wave amplitudes in a closed transmission line system to an angular spectrum of plane waves in the space system and vice versa (see figure 6) [5, pp.5-6].
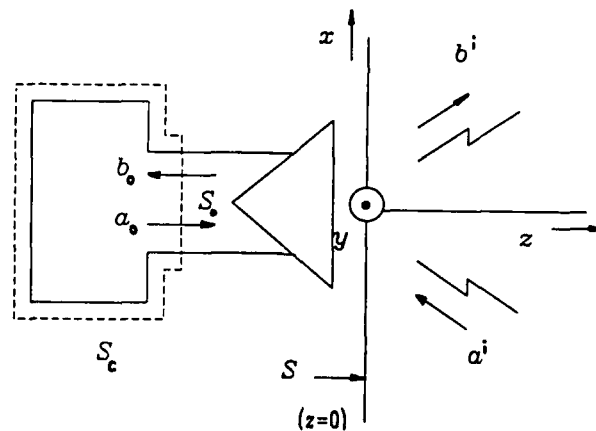
Fig.6 Antenna as multiport transducer

The scattering-matrix formulation, relating input and output quantities as defined above, is given by [1, pp.602-603; 5, p.7]:

$$b_0 = \Gamma_0 a_0 + \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \sum_{i=1}^{2} R^i(k_x,k_y) a^i(k_x,k_y) dk_x dk_y \qquad (13a)$$

$$b^i(k_x,k_y) = T^i(k_x,k_y) a_0 +$$
$$+ \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \sum_{j=1}^{2} S^{i,j}(k_x,k_y; 1_x,1_y) a^j(1_x,1_y) d1_x d1_y$$
$$\qquad (13b)$$

with i referred to as polarization index.

Γ_0 corresponds to the reflection coefficient at S_0 looking toward the antenna (see figure 1), R^i and T^i are associated with the receiving and transmitting properties of the antenna, respectively and $S^{i,j}$ describes the scattering properties. Reception from a certain direction (k_x,k_y) depends on the scattering in all directions $(1_x,1_y)$ from an object in front of the antenna. When primes are used, the same relations are true for the probe.

Now it is possible to derive a relation between the signal $b_0'(\underline{r}_0)$, received by the probe, and the input signal a_0, delivered to the test antenna. Let Γ_1' be the reflection coefficient when looking from the probe into its load, then

$$a_0' = \Gamma_1' b_0' \qquad (14)$$

To simplify the analysis, multiple reflections between AUT and probe are neglected. This means that $a^j(1_x,1_y)$ is assumed to be zero and thus from (13b):

$$b^i(k_x,k_y) = T^i(k_x,k_y) a_0 \qquad (15)$$

Substitution of (15) in (11) gives:

$$a'^i(k_x,k_y) = a_0 T^i(k_x,k_y) \exp(+j\underline{k}\cdot\underline{r}_0) \qquad (16)$$

and this equation together with (14) substituted in (13a), provided with
primes for the probe, gives:

$$b_0' - \Gamma_0' a_0' + \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \sum_{i=1}^{2} R'^i(k_x,k_y) a'^i(k_x,k_y) dk_x dk_y -$$

$$- \Gamma_0' \Gamma_1' b_0' + a_0 \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \sum_{i=1}^{2} R'^i(k_x,k_y) T^i(k_x,k_y) \cdot$$

$$\cdot \exp(+j\mathbf{k}\cdot\mathbf{r}_0) dk_x dk_y \tag{17a}$$

or:

$$b_0'(\mathbf{r}_0) - \frac{a_0}{1 - \Gamma_0' \Gamma_1'} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \sum_{i=1}^{2} R'^i(k_x,k_y) T^i(k_x,k_y) \cdot$$

$$\cdot \exp(+j\mathbf{k}\cdot\mathbf{r}_0) dk_x dk_y \tag{17b}$$

$$- \frac{a_0}{1 - \Gamma_0' \Gamma_1'} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} D(k_x,k_y) \exp(+j\mathbf{k}\cdot\mathbf{r}_0) dk_x dk_y \tag{17c}$$

with:

$$D(k_x,k_y) - \sum_{i=1}^{2} R'^i(k_x,k_y) T^i(k_x,k_y) \tag{17d}$$

$D(k_x,k_y)$ is referred to as the coupling product of the receiving and
transmitting properties of antennas. Equation (17b) is known as the
transmission equation [4, p.10].

Equation (17c) is recognized as a Fourier transform of $D(k_x, k_y)$. So the coupling product can be found by taking the Fourier inverse of (17c). Using the Fourier transform pair as stated by Papoulis [6, p.1]:

$$f(t) = (2\pi)^{-1}\int_{-\infty}^{\infty} F(\omega)\exp(j\omega t)d\omega; \quad F(\omega) = \int_{-\infty}^{\infty} f(t)\exp(-j\omega t)dt \quad (18)$$

Taking the Fourier inverse of (17c) yields:

$$D(k_x, k_x) = \frac{1 - \Gamma_0'\Gamma_1'}{4\pi^2 a_0} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} b_0'(\underline{r}_0)\exp(-j\underline{k}\cdot\underline{r}_0)dx_0 dy_0 \quad (19)$$

When the receiving characteristic $R'^i(k_x, k_y)$ ($i = 1, 2$) is known, $T^i(k_x, k_y)$ ($i = 1, 2$) can be obtained with (19) and (17d) 2). With $T^i(k_x, k_y)$ ($i = 1, 2$) the far field characteristics can be found as will be shown in a forthcoming chapter. Now some compact vector-formulations for equations derived in this paragraph will be given.

2.4 Vector formulation

The equations derived in the preceding paragraph will now be written in a more compact vector-formulation that corresponds to the formulation used in the second near-field approach and that facilitates the derivation of certain reciprocity theorems.

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2) Note that in the theory derived thusfar only one assumption is made: multiple reflections between AUT and probe are neglected. Further, due to the presence of the probe characteristic R'^i in the above equations, it is sometimes stated that probe correction of the measured results is carried out.

Equation (7) can be written as:

$$\underline{E}(\underline{r}) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \underline{b}(\underline{K}) \exp(+j\underline{k} \cdot \underline{r}) d\underline{K} \tag{20}$$

with $\underline{b}(\underline{r})$ following from (6) and (7):

$$\underline{b}(\underline{K}) = b^1(\underline{K})[\underline{K}_1 - K\gamma^{-1}\underline{a}_z] + b^2(\underline{K})\underline{K}_2 -$$
$$- b^1(\underline{K})[k/\gamma]\underline{a}_|(\underline{k}) + b^2(\underline{K})\underline{a}_\perp(\underline{k}) \tag{21}$$

The unit vectors $\underline{a}_|(\underline{k})$ and $\underline{a}_\perp(\underline{k})$ are given by [2, p.95]:

$$\underline{a}_|(\underline{k}) = \underline{K}_2 \times \underline{a}_k; \quad \underline{a}_k = \underline{k}/k \tag{22a}$$
$$\underline{a}_\perp(\underline{k}) = \underline{K}_2 \tag{22b}$$

For K<k, where $\underline{a}_|$ and $\underline{a}_\perp$ are real:

$$\underline{a}_|(\underline{k}) = \underline{a}_\theta(\underline{k}) \tag{23a}$$
$$\underline{a}_\perp(\underline{k}) = \underline{a}_\phi(\underline{k}) \tag{23b}$$

For K>k, $\underline{a}_|$ and $\underline{a}_k$ become complex, but remain unit vectors in the sense $\underline{a}_| \cdot \underline{a}_| = \underline{a}_k \cdot \underline{a}_k = 1$ [2, p.95].
In vector rotation the coupling equation becomes:

$$D(k_x, k_y) = \underline{R}'(\underline{K}) \cdot \underline{I}(\underline{K}) \tag{24}$$

It can be shown (appendix A) that for a reciprocal antenna:

$$\underline{R}'(\underline{K}) \cdot \underline{I}(\underline{K}) = -\eta_0 \eta^{-1} \underline{I}'(-\underline{K}) \cdot \underline{I}(\underline{K}); \quad \eta = \sqrt{(\epsilon/\mu)} \tag{25a}$$
$$\underline{r}'(\underline{K}) \cdot \underline{t}(\underline{K}) = [\gamma(\epsilon/\mu)^{1/2}/(\eta_0 k)]\underline{t}'(-\underline{K}) \cdot \underline{t}(\underline{K}) \tag{25b}$$

$\underline{T}(\underline{K})$ and $\underline{R}(\underline{K})$ are transverse vectors; they include only the components transverse to the z-axis. The z-component of the complete transmitting and receiving vector, can be found from the requirement that the complete vector must be orthogonal to the direction of propagation [4, p.11].

$$\underline{t}(\underline{K}) \cdot \underline{k} = 0 \tag{26a}$$

$$\underline{r}(\underline{K}) \cdot \underline{k} = 0 \tag{26b}$$

'Complete' vectors are described by small letters, transverse vectors by capitals. The reason why equation (25) is derived will become evident in the next chapters.

3 RECIPROCITY THEOREM FORMULATION APPROACH

In measuring the field of the AUT, the primary field is altered by the
probe and a true measure of the original field is, consequently, never
attained. However, probe effects can in fact be approximated *if no
mutual coupling between antennas is assumed.* It is the purpose of this
chapter to derive an expression for the probe compensation, by means of
the well-known Lorentz reciprocity theorem.

3.1 Geometrical configuration

The transmission system set-up for the planar near-field measurement
technique is shown in figure 7 [3, p.375].
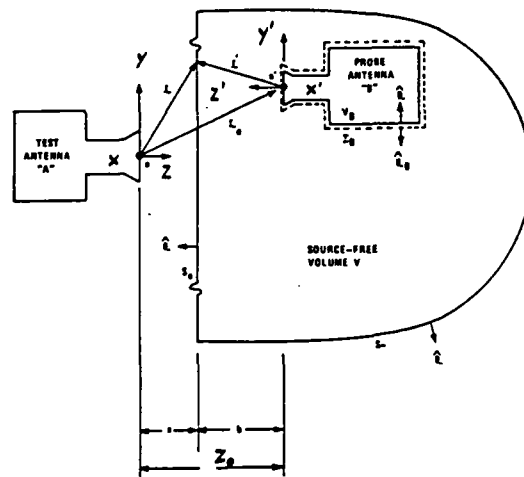
Fig. 7   Probe compensation geometry

The field generated by testantenna A is to be measured by moving the
probe antenna B over the plane surface defined by $z=z_0$.
Let the source-free volume V be bounded by the closed surface Σ,
consisting of the infinite plane S_0 at $z=a$, the surface at infinity S_∞,
and the closed surface Σ_B, surrounding the probe antenna B, operating at
the same frequency as antenna A, but not necessarily at the same time.
E_{as}, H_{as} represent the field scattered by the test antenna A, when
generator B is activated; E_{bs}, H_{bs} represent the field scattered by the
probe antenna B, when generator A is activated. The current densities
J_a, J_b, J_{as}, J_{bs} are defined correspondingly.

3.2   Lorentz reciprocity theorem

Since the volume V contains no sources, it follows from the Lorentz
reciprocity theorem that [7, pp.24-25]:

$$\iint_{\Sigma} [(E_a + E_{bs}) \times (H_b + H_{as}) - (E_b + E_{as}) \times (H_a + H_{bs})] \cdot n \, dS = 0 \qquad (27)$$

The integral vanishes identically over the surface at infinity, S_∞. This
is true because the field is a spherical TEM wave on S_∞ [8, pp.92-93]
and the integrand in (27) can be written as:

$$[k/(\omega\mu)][(E_a + E_{bs}) \times \{n \times (E_b + E_{as})\} - (E_b + E_{as}) \times \{n \times (E_a + E_{bs})\}] \cdot n \quad (28)$$

since for a TEM wave [9, p.63]:

$$H = \eta^{-1} n \times E; \quad \eta^{-1} = \sqrt{(\mu/\epsilon)} = k/(\omega\mu) \qquad (29)$$

With the vector identity $A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$, equation (28)
becomes:

$$[k/(\omega\mu)][(E_a + E_{bs}) \cdot (E_b + E_{as})\}n - \{(E_a + E_{bs}) \cdot n\}(E_b + E_{as}) +$$

$$- \{(E_b + E_{as}) \cdot (E_a + E_{bs})\}n + \{(E_b + E_{as}) \cdot n\}(E_a + E_{bs})] \cdot n \quad (30)$$

This expression vanishes identically, since $E \cdot n = 0$ on S_∞.

The surface integral over Σ_B is best evaluated by introducing the unit surface normal $n_B = -n$. If the volume V_B is linear and isotropic, the Lorentz reciprocity theorem leads to [7, pp.24-25]:

$$\iint_{\Sigma_B} [(E_a+E_{bs}) \times (H_b+H_{as}) - (E_b+E_{as}) \times (H_a+H_{bs})] \cdot (-n_B) dS =$$

$$= -\int \int \int_{V_B} [(E_b+E_{as}) \cdot (J_a+J_{bs}) - (E_a+E_{bs}) \cdot (J_b+J_{as})] dV \quad (31)$$

By definition, the scattered field E_{bs} and the currents J_a and J_{as} are zero troughout the volume V_B. To facilitate the analysis of (31), an approximation is made now: The scattered field E_{as} is neglected compared to E_b. This means that the right side of (31) can be expressed as:

$$\int \int \int_{V_B} [E_b \cdot J_{bs} - E_a \cdot J_b] dV \stackrel{\Delta}{=} P_B(r_0) \quad (32)$$

This integral is proportional to the open-circuit received voltage of the probe [3, p.376; 7, pp.94-98; 16, p.1483], thus $P_B(r_0)$ represents the measured signal within a constant of proportionality (see equation (47)).

Equation (27) can now be written as:

$$\int \int_{S_0} [(E_a+E_{bs}) \times (H_b+H_{as}) - (E_b+E_{as}) \times (H_a+H_{bs})] \cdot n dS = P_B(r_0) \quad (33)$$

In this integral, the terms involving products of primary and scattered fields vanish identically. For example,

$$\int\int_{S_0} [\underline{E}_a \times \underline{H}_{as} - \underline{E}_{as} \times \underline{H}_a] \cdot \underline{n} dS = \iint_{S_0+S_\infty} [\underline{E}_a \times \underline{H}_{as} - \underline{E}_{as} \times \underline{H}_a] \cdot \underline{n} dS =$$

$$= \int\int\int_{V+V_B} [\underline{E}_{as} \cdot \underline{J}_a - \underline{E}_a \cdot \underline{J}_{as}] dV = 0 \qquad (34)$$

The last step is true because, within V and V_B, the currents $\underline{J}_a$ and $\underline{J}_{as}$ are zero by definition; the first step is permissible because the contribution from the surface at infinity tends to zero.

Terms in (33) which involve only scattered fields will be small compared to terms only involving primary fields; hence they will be neglected. So (33) becomes:

$$\int\int_{S_0} [\underline{E}_a \times \underline{H}_b - \underline{E}_b \times \underline{H}_a] \cdot \underline{n} dS = P_B(\underline{r}_0) \qquad (35)$$

Further reduction of (35) can be effected by expanding the primary fields in terms of their wavenumber spectra over the surface S_0 (z=a).

With a time-dependence according to exp(-jωt):

$$\underline{E}_a(x,y,a) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \underline{f}(\underline{K})\exp(+j\underline{k}\cdot\underline{r})d\underline{K} \qquad (36a)$$

$$\underline{H}_a(x,y,a) = (\omega\mu)^{-1}\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \underline{k} \times \underline{f}(\underline{K})\exp(+j\underline{k}\cdot\underline{r})d\underline{K} \qquad (36b)$$

$$\underline{k}\cdot\underline{f}(\underline{K}) = 0 \qquad (36c)$$

$$\underline{E}_b(\underline{K}') = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \underline{g}(\underline{K}')\exp(+j\underline{k}'\cdot\underline{r}')d\underline{K}' \qquad (36d)$$

$$\underline{H}_b(\underline{K}') = (\omega\mu)^{-1}\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \underline{k}' \times \underline{g}(\underline{K}')\exp(+j\underline{k}'\cdot\underline{r}')d\underline{K}' \qquad (36e)$$

$$\underline{k}'\cdot\underline{g}(\underline{K}') = 0 \qquad (36f)$$

and (35) becomes:

$$(\omega\mu)^{-1}\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} [\underline{f}(\underline{K})\times(\underline{k}'\times\underline{g}(\underline{K}')) - \underline{g}(\underline{K}')\times(\underline{k}\times\underline{f}(\underline{K}))] \cdot$$

$$(-\underline{a}_z)\exp(+j\underline{k}\cdot\underline{r} + j\underline{k}'\cdot\underline{r}')d\underline{K}'d\underline{K}dxdy \qquad (37)$$

The evaluation of (37) is rather involved and will be presented in appendix B [10]. It appears that (37) can be written as:

$$[4\pi^2/(\omega\mu)] \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} [\underline{f}(\underline{K}) \times (\underline{k} \times \underline{g}(-\underline{K})) + \underline{g}(-\underline{K}) \times (\underline{k} \times \underline{f}(\underline{K}))] \cdot \underline{a}_z \exp(+j\underline{k} \cdot \underline{r}_0) -$$

$$- P_B(\underline{r}_0) \qquad (38)$$

Using the vector identity $\underline{A} \times (\underline{B} \times \underline{C}) - (\underline{A} \cdot \underline{C})\underline{B} - (\underline{A} \cdot \underline{B})\underline{C}$ and equations (36c,f) yields:

$$[8\pi^2/(\omega\mu)] \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} k_z \underline{f}(\underline{K}) \cdot \underline{g}(-\underline{K}) \exp(+j\underline{k} \cdot \underline{r}_0) d\underline{K} - P_B(\underline{r}_0) \qquad (39)$$

Equation (39) is recognized as a two-dimensional Fourier transform. Taking the inverse over the plane $z=z_0$ gives (see equation (18)):

$$\underline{f}(\underline{K}) \cdot \underline{g}(-\underline{K}) - \frac{1}{(2\pi)^2} \frac{\omega\mu}{8\pi^2} \frac{1}{k_{z_\infty}} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} P_B(\underline{r}_0) \exp(-j\underline{k} \cdot \underline{r}_0) dx_0 dy_0 \qquad (40)$$

4 EQUIVALENCE OF BOTH APPROACHES

From inspection of equations (20) and (36) for the scattering-matrix
formulation approach and the reciprocity theorem formulation approach,
respectively:

$$\underline{E}(\underline{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{b}(\underline{K}) \exp(+j\underline{k} \cdot \underline{r}) d\underline{K} \qquad (41a)$$

Scattering-Matrix Formulation Approach

$$\underline{E}(\underline{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{f}(\underline{K}) \exp(+j\underline{k} \cdot \underline{r}) d\underline{K} \qquad (41b)$$

Reciprocity Theorem Formulation Approach

it follows that

$$\underline{f}(\underline{K}) \cdot \underline{g}(-\underline{K}) = \underline{b}(\underline{K}) \cdot \underline{b}'(-\underline{K}) \qquad (42a)$$

because:

$$\underline{b}(\underline{K}) = 4\pi^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{E}(\underline{r}) \exp(-j\underline{k} \cdot \underline{r}) dx dy \qquad (42b)$$

$$= \underline{f}(\underline{K})$$

With (15) is found:

$$\underline{b}(\underline{K}) - a_0 \underline{t}(\underline{K}) \tag{43}$$

since:

$$\underline{b}(\underline{K}) - b^1(\underline{K})[k/\gamma]\underline{a}_|(\underline{k}) + b^2(\underline{K})\underline{a}\perp(\underline{k}) \tag{21}$$

$$\underline{t}(\underline{K}) - T^1(\underline{K})[k/\gamma]\underline{a}_|(\underline{k}) + T^2(\underline{K})\underline{a}\perp(\underline{k}) \tag{A.30}$$

and so (42a) becomes:

$$\underline{f}(\underline{K}) \cdot \underline{g}(-\underline{K}) - a_0^2 \underline{t}(\underline{K}) \cdot \underline{t}'(-\underline{K}) \tag{44}$$

Equation (44) substituted in (39) gives (with $k_z \in R$):

$$P_B(\underline{r}_0) - \frac{8\pi^2 a_0^2}{\omega\mu} k\cos\theta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{t}(\underline{K}) \cdot \underline{t}'(-\underline{K})\exp(+j\underline{k}\cdot\underline{r}_0)d\underline{K} \tag{45}$$

Equation (25b) gives:

$$\underline{t}'(-\underline{K}) - \frac{\eta_0 k}{\gamma\sqrt{(\epsilon/\mu)}} \underline{r}'(\underline{K}); \quad k_z - \gamma \tag{46}$$

and this substituted in (45) with $k \simeq \omega\sqrt{(\epsilon\mu)}$ gives:

$$P_B(\underline{r}_0) = 8\pi^2 a_0^2 \eta_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{r}'(\underline{K}) \cdot \underline{t}(\underline{K}) \exp(+j\underline{k}\cdot\underline{r}_0) d\underline{K} =$$

$$= 8\pi^2 a_0^2 \eta_0 \frac{1 - \Gamma_0'\Gamma_1'}{a_0} \left\{ \frac{a_0}{1 - \Gamma_0'\Gamma_1'} \cdot \right.$$

$$\left. \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{r}'(\underline{K}) \cdot \underline{t}(\underline{K}) \exp(+j\underline{k}\cdot\underline{r}) d\underline{K} \right\} =$$

$$= 8\pi^2 a_0 \eta_0 (1 - \Gamma_0'\Gamma_1') b_0'(\underline{r}_0) \qquad (47)$$

with $b_0'(\underline{r}_0)$ given in (17b,c).
Substitution of (44), (46) and (47) in (40) gives:

$$a_0^2 \frac{\eta_0 k}{\gamma\sqrt{(\epsilon/\mu)}} \underline{t}(\underline{K}) \cdot \underline{r}'(\underline{K}) = \frac{1}{(2\pi)^2} \frac{\omega\mu}{8\pi^2} \frac{1}{k_z} 8\pi^2 a_0 \eta_0 (1 - \Gamma_0'\Gamma_1') \cdot$$

$$\cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b_0'(\underline{r}_0) \exp(-j\underline{k}\cdot\underline{r}_0) dx_0 dy_0 \qquad (48)$$

or, with (24):

$$D(k_x, k_y) = \frac{\omega\mu\sqrt{(\epsilon/\mu)}}{k(2\pi)^2} \frac{(1 - \Gamma_0'\Gamma_1')}{a_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b_0'(\underline{r}_0) \exp(-j\underline{k}\cdot\underline{r}_0) dx_0 dy =$$

$$= \frac{1 - \Gamma_0'\Gamma_1'}{4\pi^2 a_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b_0'(\underline{r}_0) \exp(-j\underline{k}\cdot\underline{r}_0) dx_0 dy_0 \qquad (49)$$

with $D(k_x,k_y) = \underline{t}(\underline{K})\cdot\underline{r}'(\underline{K}) = \underline{T}(\underline{K})\cdot\underline{R}'(\underline{K})$.

This is the same result as stated in equation (19) and, consequently, both approaches are equivalent.

5 FAR-FIELD ANTENNA CHARACTERISTICS

Now that is proven that both approaches lead to the same results, a choice in favour of one of the two approaches has to be made. Chosen is for the scattering-matrix formulation approach. Maybe this approach is a little less transparent than the reciprocity theorem approach, but it seems that the scattering-matrix formulation approach is the most widely adopted one in the area of planar near-field analysis [11], obviously due to the pioneering work done and expertise available at the U.S. National Bureau of Standards (NBS).

In the remainder of this chapter, equations for the far-field, power-gain function and receiving area of an Antenna Under Test will be derived.

5.1 Far-field of Antenna Under Test

The starting point for the derivation of the far-field formulation is equation (20):

$$\mathbf{E}(\mathbf{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{b}(\mathbf{K}) \exp(+j\mathbf{k} \cdot \mathbf{r}) d\mathbf{K} \tag{50}$$

With equation (43):

$$\mathbf{E}(\mathbf{r}) = a_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{t}(\mathbf{K}) \exp(+j\mathbf{k} \cdot \mathbf{r}) d\mathbf{K} \tag{51}$$

Normalized to the input signal (a_0=1), the E-field becomes:

$$\underline{E}(\underline{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{t}(\underline{K}) \exp(+j\underline{k}\cdot\underline{r}) d\underline{K} \tag{52}$$

By the method of the steepest descent (known also as the saddle point method of integration), or, alternatively, the method of stationary phase, (52) can be approximated in the far field, that is for large r. As proven in appendix C by the last mentioned method [12, pp.28-38], the far-field formulation is given by:

$$\underset{r\to\infty}{\underline{E}(\underline{r})} \approx -j2\pi k_{z0}\underline{t}(\underline{K})\frac{e^{+jkr}}{r}; \qquad k_{z0} = k\frac{z}{r}$$

$$= -j2\pi k\cos\theta\,\underline{t}(\underline{K})\frac{e^{+jkr}}{r}; \qquad k_{z0}\in R \tag{53}$$

This equation differs from the far-field given by Newell[4, p.19] by a factor 2π, due to an applied normalization that cancels out this factor [2, pp.271-274]. The equation as stated above is also found in [17, p.11] and will be used in the remainder of this report.

5.2   Power-gain function

The power-gain function is given by [8, p.43]:

$$\text{Gain} = 4\pi\frac{\text{radiation intensity}}{\text{total input power}} = 4\pi\frac{U(\theta,\phi)}{P_{in}} \tag{54a}$$

in which the radiation intensity is given by [8, p.28]:

$$U(\theta,\phi) = \frac{r^2}{2} \eta \, |E(r)|^2; \quad \eta = \sqrt{(\epsilon/\mu)} \tag{54b}$$

η is the characteristic *admittance* of the medium.

The total input power P_{in} is given by [13, p.536]:

$$P_{in} = 1/2 \, Re\{V_0 I_0^*\} \tag{55}$$

With [14, p.24]:

$$V_0 = a_0 + b_0 \tag{56a}$$
$$I_0 = \eta_0(a_0 - b_0) \tag{56b}$$

η_0 is the characteristic *admittance* of the antenna. So the input power is:

$$\begin{aligned}
P_{in} &= 1/2 \, \eta_0 Re\{(a_0 + b_0)(a_0^* - b_0^*)\} = \\
&= 1/2 \, \eta_0 Re\{a_0 a_0^* - b_0 b_0^* + a_0^* b_0 - a_0 b_0^*\} = \\
&= 1/2 \, \eta_0(|a_0|^2 - |b_0|^2) \tag{57}
\end{aligned}$$

since $a_0^* b_0 - a_0 b_0^*$ is pure imaginary.

With $b_0 = \Gamma_0 a_0$ (Γ_0 is the reflection coefficient when looking into the antenna):

$$P_{in} = 1/2 \, \eta_0 |a_0|^2 (1 - |\Gamma_0|^2) \tag{58}$$

The power gain is found by substituting (53), (54b) and (58) in (54a) with $a_0 = 1$ (because of the normalization of equation (51)).

For the gain is found (see also [1, p.606]):

$$G(\mathbf{K}) = 16\pi^3 k^2 \cos^2\theta \frac{|\mathbf{t}(\mathbf{K})|^2 \eta}{(1 - |\Gamma_0|^2)\eta_0}; \quad \eta = \sqrt{(\epsilon/\mu)} \tag{59}$$

This equation differs from that given by Newell[4, p.19], due to the 2π difference in the far-field equation.

5.3 Receiving effective area

The receiving effective area of an antenna, that is smaller than the physical aperture, is related to the power-gain according to [15, pp.178-179]:

$$\sigma(\mathbf{K}) = [\lambda^2/(4\pi)]G(\mathbf{K}); \quad \lambda = 2\pi/k \tag{60}$$

To express the *receiving effective area* totally in receiving characteristics, use is made of equation (46), and together with (59) and (60) this yields:

$$\sigma(\mathbf{K}) = \frac{\lambda^2 16\pi^3 k^2 \cos^2\theta |\mathbf{r}'(-\mathbf{K})|^2 \eta_0^2 k^2 \eta}{4\pi(1 - |\Gamma_0|^2)\eta_0 \gamma^2 \eta^2} =$$

$$= 16\pi^4 \frac{|\mathbf{r}'(-\mathbf{K})|^2 \eta_0}{(1 - |\Gamma_0|^2)\eta}; \quad \eta = \sqrt{(\epsilon/\mu)} \tag{61}$$

The difference between the equation (61) and that stated by Newell [4, p.19] is caused by the 2π-difference in the far-field equation.

6   RESEARCH ITEMS

Now, that the theory of near-field measurement is understood, attention
can be paid to the following items:

- co - and crosspolar reception: The elctric field vectors of AUT
and probe can be split in two orthogonal components. In one
measurement the probe will couple primarily to one of the
components, called the copolar component, the other component is
referred to as the crosspolar component. In a second measurement
with the probe 90 degrees rotated, it will couple primarily to
the other component, then referred to as copolar component. In
order to obtain the desired components in two measurements, first
an appropriate definition of crospolarization has to be stated;

- sampling algorithm: First the distance between scanning plane and
probe has be chosen. Then the sample spacing and the number of
sample points must be selected to include the details of the
field in the aperture;

- measuring active antennas in transmitting and receiving mode: In
the theory derived before it was assumed that the AUT is
operating as transmitting antenna and the probe as receiving
antenna. For measuring an antenna in the receiving mode, these
roles have to change. It is very likely that the theory, with
some little modifications, will hold true for this situation;

- error analysis: In order to draw conclusions from the results
obtained from near-field measurements, a thorough understanding
of possible error sources and their influence on the total
results must be available. Of the three possible error
categories: theory, numerical calculations, and measurement, the
latter will be responsible for most of the errors involved;

- pulsed measurements: The consequences of pulsed measurements for
the near-field theory, that is derived under the assumption of
unmodulated Continuous Wave operation, have to be investigated.

7   CONCLUSIONS

A successful near-field antenna measurements program requires, besides
careful measurements and computer processing of the data, a reasonable
understanding of the theory involved.
This theory has been covered in detail in this report, following two
approaches, and can be used as a guide in the understanding of the
theory of planar near-field measurement.

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APPENDIX A   RECIPROCITY

To prove is:

$$\eta_0 k\underline{r}'(\underline{K}) \sim \eta\gamma\underline{t}'(-\underline{K}); \quad \eta - \sqrt{(\epsilon/\mu)} \tag{A.1}$$

In order to show the validity of (A.1), first the following so-called reciprocity lemma is proven:

$$\eta_0(a_0'b_0 - a_0b_0') - \int_{\underline{K}} [a'(\underline{K})b(\underline{K}) - a(\underline{K})b'(\underline{K})]\eta(\underline{K})d\underline{K} \tag{A.2}$$

A.1   Reciprocity lemma (relationship between transverse receiving
           and transmitting characteristics)

Equation (A.2) relates the electromagnetic fields on the AUT-aperture to the electromagnetic fields on the probe-aperture. To prove (A.2), first the field description on an arbitrary aperture S, perpendicular to the z-axis (see figure 1), is given.

A.1.1   Transverse field
The electric and magnetic fields $\underline{E}_t$ and $\underline{H}_t$ must satisfy the following Maxwell equations on S (an $\exp(-j\omega t)$ time-dependence is assumed):

$$\nabla \times \underline{E}_t - j\omega\mu\underline{H}_t \tag{A.3a}$$
$$\nabla \times \underline{H}_t - -j\omega\epsilon\underline{E}_t \tag{A.3b}$$

In the rectangular coordinate system xyz of figure 1, the ∇-operator can be divided into a transverse and an axial part according to [A1, p.67]:

$$\nabla_t = \underline{a}_x \, \delta/\delta x + \underline{a}_y \, \delta/\delta y \tag{A.4a}$$

$$\nabla_z = \underline{a}_z \, \delta/\delta z \tag{A.4b}$$

With (A.4) and the fact that $\underline{E}_t$ and $\underline{H}_t$ do not possess a z-component, (A.1) can be written as:

$$\underline{a}_z \times \delta/\delta z \, \underline{E}_t = j\omega\mu\underline{H}_t \tag{A.5a}$$

$$\underline{a}_z \times \delta/\delta z \, \underline{H}_t = -j\omega\epsilon\underline{E}_t \tag{A.5b}$$

$$\nabla_t \times \underline{E}_t = \underline{0} \tag{A.5c}$$

$$\nabla_t \times \underline{H}_t = \underline{0} \tag{A.5d}$$

The last two equations indicate that $\underline{E}_t$ and $\underline{H}_t$ can be written as, respectively [A1, p.68]:

$$\underline{E}_t = g_1(z)\nabla_t\Phi \tag{A.6a}$$

$$\underline{H}_t = g_2(z)\nabla_t\Psi \tag{A.6b}$$

Equations (A.3a,b) implicate:

$$\delta^2/\delta z^2 \, \underline{E}_t + k^2\underline{E}_t = \underline{0}; \quad k^2 = \omega^2\epsilon\mu \tag{A.7}$$

and with (A.6a) substituted in (A.7), it is found that $g_1(z)$ can be written as:

$$g_1(z) = A_\pm \exp(\pm jkz) \tag{A.8}$$

So for the electric field is found:

$$\underline{E}_t = A_{\pm} \nabla_t \Phi(x,y) \exp(\pm jkz) \tag{A.9a}$$

and with (A.5a):

$$\underline{H}_t = \mp \eta \underline{a}_z \times A_{\pm} \nabla_t \Phi \exp(\pm jkz); \quad \eta = \sqrt{(\epsilon/\mu)} \tag{A.9b}$$

A.1.2   Two-port description of AUT-probe system

The Aut-probe system is _modelled_ as a two port, see figure A1

Fig. A1   Two-port model of Aut-probe system

Incident and reflected voltage waves at port n (n=1,2) are given, in
amplitude and phase, by V_n^+ and V_n^-, respectively. Incident and
reflected waves are normalized according to Kerns & Beatty [A2, p.24]
A1)

$$a_n = V_n^+; \quad b_n = V_n^- \qquad\qquad\qquad (A.10a)$$

A1) The normalization is somewhat arbitrary. The results are normali-
    zation-independent, as long as the normalization is such that
    $I/V = \eta$ for an incident or reflected wave [A.5, pp.525-526].

Voltage and current on the reference planes are given by:

$$V_n = V_n^+ + V_n^- = (a_n + b_n) \tag{A.10b}$$

$$I_n = C_n(V_n^+ - V_n^-) = C_n(a_n - b_n) \tag{A.10c}$$

V_n and I_n are introduced to facilitate the calculations; they are not available for measurement.

The characteristic admittance of the nth port is connected in a simple way with C_n [A.6, pp.146-147]. To show this connection, let $b_n=0$; then:

$$V_n = a_n \tag{A.11a}$$

$$I_n = C_n a_n \tag{A.11b}$$

$$\eta_n = I_n/V_n = C_n \tag{A.11c}$$

When the antennas are matched to free-space:

$$C_n = \eta_n = \eta \tag{A.11d}$$

The above is now applied to the theory in paragraph A.1.1 by relating the voltage to the electric field and the current to the magnetic field:

$$\mathbf{E}_t = V_n \nabla_t \Phi \tag{A.12a}$$

$$\mathbf{H}_t = I_n \nabla_t \Psi \tag{A.12b}$$

Substitution of (A.10) and (A.11) in (A.12) gives:

$$\mathbf{E}_{tn} = (a_n + b_n)\nabla_t \Phi = (a_n + b_n)\mathbf{e}_n \tag{A.13a}$$

$$\mathbf{H}_{tn} = (a_n - b_n)\eta \nabla_t \Psi = (a_n - b_n)\mathbf{h}_n \tag{A.13b}$$

(A.9) yields:

$$\underline{H}_{tn} = \eta \underline{a}_z \times \underline{E}_{tn} \tag{A.14}$$

or, with (A.13):

$$\underline{h}_n = \eta \frac{a_n + b_n}{a_n - b_n} \underline{a}_z \times \underline{e}_n \tag{A.15}$$

Substitution of (A.10) and (A.11d) in (A.15) gives:

$$\underline{h}_n = \eta \frac{V_n}{I_n/\eta} \underline{a}_z \times \underline{e}_n$$

$$= \eta^2 \frac{V_n}{I_n} \underline{a}_z \times \underline{e}_n =$$

$$= \eta^2 \eta^{-1} \underline{a}_z \times \underline{e}_n =$$

$$= \eta \underline{a}_z \times \underline{e}_n \tag{A.16}$$

A.1.3   Reciprocity theorem

Consider the two-antenna system of figure A2 [A3, p.95]
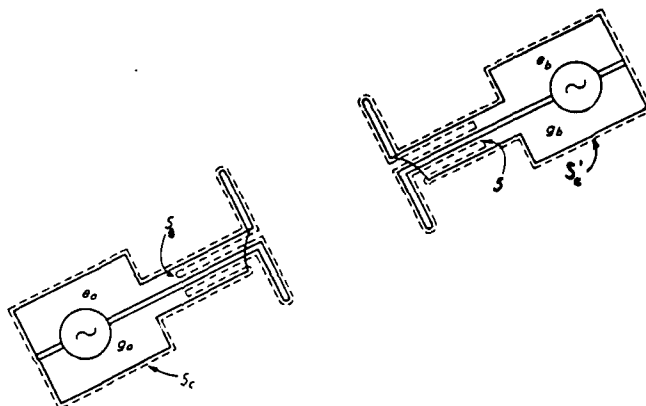
Fig. A2   Two-antenna system

In the source-free region, bounded by the surfaces S, S_0, S_c, S_c' and S_∞ the following Lorentz reciprocity theorem is valid:

$$\int\int_\Sigma (\mathbf{E} \times \mathbf{H}' - \mathbf{E}' \times \mathbf{H}) \cdot \mathbf{n} dS = 0 \qquad (A.17)$$

Σ consists of the (perfect) conducting surfaces S_c and S_c', the radiating surfaces S and S_0 and the surface at infinity S_∞.

The contributions from S_c and S_c' to (A.17) are zero, because $\underline{E}_t = \underline{0}$ on S_c and S_c'. On S_∞ is $\underline{H} = \eta \underline{a}_r \times \underline{E}$, and, consequently:

$$\int \int_{S_\infty} (\underline{E} \times \underline{H}' - \underline{E}' \times \underline{H}) \cdot \underline{n} dS = \eta \int \int_{S_\infty} (\underline{E} \times \underline{a}_r \times \underline{E}' - \underline{E}' \times \underline{a}_r \times \underline{E}) \cdot \underline{a}_r dS =$$

$$= \eta \int \int_{S_\infty} [\underline{a}_r (\underline{E} \cdot \underline{E}') - \underline{E}' (\underline{E} \cdot \underline{a}_r) - \underline{a}_r (\underline{E}' \cdot \underline{E}) + \underline{E} (\underline{E}' \cdot \underline{a}_r)] \cdot \underline{a}_r dS =$$

$$= 0 \qquad\qquad\qquad\qquad (A.18)$$

So for the system of figure A2, (A.17) results in:

$$\int \int_{S_0} (\underline{E}_t \times \underline{H}_t' - \underline{E}_t' \times \underline{H}_t) \cdot -\underline{a}_z dS = \int \int_{S} (\underline{E}_t' \times \underline{H}_t - \underline{E}_t \times \underline{H}_t') \cdot \underline{a}_z dS \quad (A.19)$$

The primes in this equation are associated with the Probe (see figure A3).
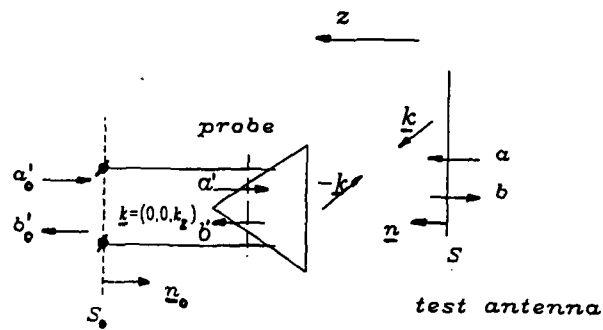
Fig. A3   Testantenna-probe system

On the antenna apertures S and S_0, the fields $\underline{E}_t$ and $\underline{H}_t$ only depend on the transverse part of $\underline{k}$ ($\underline{K}$) and the transverse coordinates (x,y). The integrand on (A.19) can be written as (with reference to figure A3):

$$\underline{E}_{tn} \times \underline{H}_{tn}' \cdot \underline{E}_{tn}' \times \underline{H}_{tn} = \int_{\underline{K}} \Big\{ [a_n(\underline{K}) + b_n(\underline{K})][a_n'(-\underline{K}) - b_n'(-\underline{K})](\underline{e}_n \times \underline{h}_n') +$$

$$- [a_n'(-\underline{K}) + b_n'(-\underline{K})][a_n(\underline{K}) - b_n(\underline{K})](\underline{e}_n' \times \underline{h}_n) \Big\} d\underline{K}$$

$$\overset{(A.15)}{=} \int_{\underline{K}} \eta(\underline{K}) \Big\{ [a_n(\underline{K})a_n'(-\underline{K}) - b_n(\underline{K})b_n'(-\underline{K}) + a_n'(-\underline{K})b_n(\underline{K}) - a_n(\underline{K})b_n'(-\underline{K})] \cdot$$

$$\cdot (\underline{e}_n \times \underline{a}_z \times \underline{e}_n') +$$

$$- [a_n(\underline{K})a_n'(-\underline{K}) - b_n(\underline{K})b_n'(-\underline{K}) - a_n'(-\underline{K})b_n(\underline{K}) + a_n(\underline{K})b_n'(-\underline{K})] \cdot$$

$$\cdot (\underline{e}_n' \times \underline{a}_z \times \underline{e}_n) \Big\} d\underline{K} =$$

$$= \int_{\underline{K}} \eta(\underline{K}) \Big\{ [a_n(\underline{K})a_n'(-\underline{K}) - b_n(\underline{K})b_n'(-\underline{K}) + a_n'(-\underline{K})b_n(\underline{K}) - a_n(\underline{K})b_n'(-\underline{K})] \cdot$$

$$\cdot (\underline{a}_z(\underline{e}_n \cdot \underline{e}_n') - \underline{e}_n'(\underline{e}_n \cdot \underline{a}_z)) +$$

$$- [a_n(\underline{K})a_n'(-\underline{K}) - b_n(\underline{K})b_n'(-\underline{K}) + a_n(\underline{K})b_n'(-\underline{K}) - a_n'(-\underline{K})b_n(\underline{K})] \cdot$$

$$\cdot (\underline{a}_z(\underline{e}_n' \cdot \underline{e}_n) - \underline{e}_n(\underline{e}_n' \cdot \underline{e}_n)) \Big\} d\underline{K} =$$

$$= 2\underline{a}_z \int_{\underline{K}} \eta(\underline{K})[a_n'(-\underline{K})b(\underline{K}) - a_n(\underline{K})b_n'(-\underline{K})][\underline{e}_n \cdot \underline{e}_n'] d\underline{K} \qquad (A.20)$$

The dot product $[\underline{e}_n \cdot \underline{e}_n']$ in the integrand of (A.20) illustrates the influence of the polarization.

Returning to (A.19), taking reference plane S_0 *in* the waveguide behind the probe, where $\underline{K} - \underline{0}$ (see figure A3) and taking reference plane S *on* the AUT aperture, equation (A.19) becomes:

$$\int\int_{S_0} \eta_0 [a_0'b_0 - a_0b_0'][\underline{e}_0 \cdot \underline{e}_0']dx_0dy_0 - \int\int_S \left\{ \int_{\underline{K}} [a'(-\underline{K})b(\underline{K}) + \right.$$

$$\left. - a(\underline{K})b'(-\underline{K})]\eta(\underline{K})] \right\} [\underline{e} \cdot \underline{e}']dxdy \qquad (A.21)$$

or:

$$\eta_0(a_0'b_0 - a_0b_0') - \frac{\displaystyle\int\int_S \underline{e} \cdot \underline{e}'\,dxdy}{\displaystyle\int\int_{S_0} \underline{e}_0 \cdot \underline{e}_0'\,dx_0dy_0} \int_{\underline{K}} [a'(-\underline{K})b(\underline{K}) - a(\underline{K})b'(-\underline{K})]\eta(\underline{K})d\underline{K}$$

$$(A.22)$$

Including both polarizations:

$$\eta_0(a_0'b_0 - a_0b_0') - \frac{\displaystyle\int\int_S \underline{e} \cdot \underline{e}'\,dxdy}{\displaystyle\int\int_{S_0} \underline{e}_0 \cdot \underline{e}_0'\,dx_0dy_0} \int_{\underline{K}} [\underline{a}'(-\underline{K}) \cdot \underline{b}(\underline{K}) - \underline{a}(\underline{K}) \cdot \underline{a}'(-\underline{K})]\eta(\underline{K})d\underline{K}$$

$$(A.23)$$

(Note the dot product).

When the medium is isotropic, η is independent of $\underline{K}$ and can be placed outside the integral.

Equation (A.23) is also valid for the situation wherein AUT and probe
are interchanged, with reference planes on the probe aperture and in the
waveguide behind the AUT.

A.1.4   Antenna reciprocity

To derive an equation relating the transverse receiving characteristic
$\underline{R}(\underline{K})$ to the transverse transmission characteristic $\underline{T}(\underline{K})$, use is made of
equations (13a,b).

Scattering equations for AUT and probe are given by:

$$b_0 = \Gamma_0 a_0 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{i=1}^{2} R^i(\underline{K}) a^i(\underline{K}) d\underline{K} \tag{A.24a}$$

$$b^i(\underline{K}) = T^i(\underline{K}) a_0 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{j=1}^{2} S^{i,j}(\underline{K},\underline{L}) a^j(\underline{L}) d\underline{L} \tag{A.24b}$$

$$b_0' = \Gamma_0' a_0' + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{i=1}^{2} R'^i(\underline{K}) a'^i(\underline{K}) d\underline{K} \tag{A.24c}$$

$$b'^i(\underline{K}) = T'^i(\underline{K}) a_0' + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{j=1}^{2} S'^{i,j}(\underline{K},\underline{L}) a'^j(\underline{L}) d\underline{L} \tag{A.24d}$$

With a proper choice of excitation the wanted equation can be derived.
Consider the excitation:

$$a_0 = 1; \quad a^i(\underline{K}) = 0$$
$$\tag{A.25}$$
$$a_0' = 0; \quad a'^j(\underline{K}) = \delta_{ij}\delta(\underline{K}-\underline{L})$$

This means: The AUT is transmitting and the probe is receiving *from the
direction of the AUT*, see also figure 6, page 11.

With (A.22) is found:

$$-\eta_0 b_0' = \eta \frac{\displaystyle\int\int_S \underline{e}\cdot\underline{e}'\,dxdy}{\displaystyle\int\int_{S_0} \underline{e}_0\cdot\underline{e}_0'\,dx_0 dy_0} \int_{\underline{K}} [a'^i(-\underline{K})b^i(\underline{K})]d\underline{K} -$$

$$-\eta \frac{\displaystyle\int\int_S \underline{e}\cdot\underline{e}'\,dxdy}{\displaystyle\int\int_{S_0} \underline{e}_0\cdot\underline{e}_0'\,dx_0 dy_0} \int_{\underline{L}} [a'^j(-\underline{L})b^j(\underline{L})]d\underline{L} -$$

$$-\eta \frac{\displaystyle\int\int_S \underline{e}\cdot\underline{e}'\,dxdy}{\displaystyle\int\int_{S_0} \underline{e}_0\cdot\underline{e}_0'\,dx_0 dy_0} \int_{\underline{L}} [\delta_{ij}\delta(-\underline{L}-\underline{K})b^j(\underline{L})]d\underline{L} -$$

$$-\eta \frac{\displaystyle\int\int_S \underline{e}\cdot\underline{e}'\,dxdy}{\displaystyle\int\int_{S_0} \underline{e}_0\cdot\underline{e}_0'\,dx_0 dy_0} b^i(-\underline{K}) \qquad\qquad (A.26a)$$

since:

$$\delta_{ij} = \begin{cases} 1 & \text{for } i=j \\ 0 & \text{else} \end{cases} \qquad\qquad (A.26b)$$

$$\delta(-\underline{L}-\underline{K}) = \begin{cases} 1 & \text{for } \underline{L}=-\underline{K} \\ 0 & \text{else} \end{cases} \qquad\qquad (A.26c)$$

With the excitation according to (A.25), (A.24) yields:

$$b^1(\underline{K}) - T^1(\underline{K}) \qquad\qquad\qquad (A.27a)$$
$$b_0' - R'^1(\underline{K}) \qquad\qquad\qquad (A.27b)$$

so (A.26a) transforms to:

$$-\eta_0 R'^1(\underline{K}) - \eta \; \frac{\displaystyle\int\int_S \underline{e}\cdot\underline{e}'\,dxdy}{\displaystyle\int\int_{S_0} \underline{e}_0\cdot\underline{e}_0'\,dx_0dy_0} \; T^1(-\underline{K}) \qquad\qquad (A.28)$$

When both antennas are identical, the wanted equation is found to be:

$$\eta_0 \underline{R}(\underline{K}) - -\eta \underline{T}(-\underline{K}) \qquad\qquad\qquad (A.29)$$

since S and S_0 are the same in magnitude, see figure A4. It is obvious that $S - S_0$ when the probe is a waveguide. For a horn antenna (see dashed line in figure A4), the fields at the aperture can be found by treating the horn as a radial waveguide (dotted line in figure A4) radiating the dominant mode of the waveguide-feed [A7, pp.533-534; A8, pp.349], so also in this case $S - S_0$.

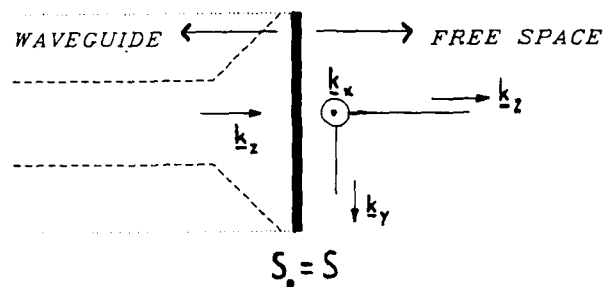WAVEGUIDE ← ——→ FREE SPACE

$S_o = S$

Fig.A4   Antenna aperture

A.2   Relationship between complete transmitting and receiving
         characteristic

In analogy with (21) (that is related to a transmitted wave), the
complete transmitting characteristic is given by [A9, p.12]:

$$\underline{t}(\underline{K}) = [k/\gamma]T^1(\underline{K})\underline{a}_{\parallel}(\underline{k}) + T^2(\underline{K})\underline{a}_{\perp}(\underline{k}) \qquad (A.30)$$

In analogy with (7), that gives the transmitted field of an antenna, the
received field is given by:

$$\underline{E}(\underline{r}) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} [a^1(\underline{K})\underline{E}_1{}^-(\underline{K},\underline{r}) + a^2(\underline{K})\underline{E}_2{}^-(\underline{K},\underline{r})]dk_x dk_y \qquad (A.31)$$

With (6a) and (6c) the integrand can be written as:

$$a^1(\underline{K})\underline{E}_1^-(\underline{K},\underline{r}) + a^2(\underline{K})\underline{E}(\underline{K},\underline{r}) =$$
$$= [a^1(\underline{K})\{\underline{K}_1 + k\gamma^{-1}\underline{a}_z\} + a^2(\underline{K})\underline{K}_2]\exp(+j\underline{k}\cdot\underline{r}) \qquad (A.32)$$

This means that the received transverse field $\underline{E}_t(\underline{r})$ is given by [2, p.91]:

$$\underline{E}_t(\underline{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [a^1(\underline{K})\underline{K}_1 + a^2(\underline{K})\underline{K}_2]\exp(+j\underline{k}\cdot\underline{r})dk_xdk_y =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{A}(\underline{K})\exp(+j\underline{k}\cdot\underline{r})dk_xdk_y \qquad (A.33a)$$

with:

$$\underline{A}(\underline{K}) = a^1(\underline{K})\underline{K}_1 + a^2(\underline{K})\underline{K}_2 \qquad (A.33b)$$

Now assume that reception from only *one* direction takes place, then:

$$\underline{E}_t(\underline{r}) = \underline{A}(\underline{K})\exp(+j\underline{k}\cdot\underline{r}) \qquad (A.34)$$

With (13a):

$$b_0 = \Gamma_0 a_0 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{i=1}^{2} R^i(\underline{K}) a^i(\underline{K}) dk_x dk_y =$$

$$= \Gamma_0 a_0 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{R}(\underline{K}) \cdot \underline{A}(\underline{K}) dk_x dk_y =$$

$$= \Gamma_0 a_0 + \underline{R}(\underline{K}) \cdot \underline{A}(\underline{K}) \qquad\qquad (A.35)$$
$$\textit{reception from one direction}$$

with $\underline{A}(\underline{K})$ as defined above and:

$$\underline{R}(\underline{K}) = R^1(\underline{K})\underline{K}_1 + R^2(\underline{K})\underline{K}_2 \qquad\qquad (A.36)$$

To avoid confusion with vector $\underline{A}$ as used in paragraph 2.2, $\underline{A}(\underline{K})$ in (A.35) will be rewritten $\underline{C}(\underline{K})$:

$$b_0 = \Gamma_0 c_0 + \underline{R}(\underline{K}) \cdot \underline{C}(\underline{K}) \qquad\qquad (A.37a)$$
$$\underline{C}(\underline{K}) = c^1(\underline{K})\underline{K}_1 + c^2(\underline{K})\underline{K}_2 \qquad\qquad (A.37b)$$

Vector $\underline{c}$ is given by (compare with (21)   ):

$$\underline{c}(\underline{K}) = c^1(\underline{K})[\underline{K}_1 + K\gamma^{-1}\underline{a}_z] + c^2(\underline{K})\underline{K}_2; \quad k_z = -\gamma \qquad (A.38)$$

$k_z = -\gamma$ because the wave is *incident* upon the antenna.

With (5) and (22):

$$\underline{a}_|(\underline{k}) = [1/kK](\underline{a}_z \times \underline{K}) \times \underline{k} =$$
$$= -[1/kK]\underline{k} \times (\underline{a}_z \times \underline{K}) =$$
$$= -[1/kK]\{\underline{a}_z(\underline{k} \cdot \underline{K}) - \underline{K}(\underline{k} \cdot \underline{a}_z)\} =$$
$$= -[1/kK]\{K^2 \underline{a}_z + \gamma \underline{K}\} =$$
$$= -\{[K/k]\underline{a}_z + [\gamma/k][\underline{K}/K]\} =$$
$$= -[\gamma/k]\{\underline{K}_1 + K\gamma^{-1}\underline{a}_z\} \qquad (A.39)$$

and thus equation (A.38) becomes:

$$\underline{c}(\underline{K}) = -[k/\gamma]c^1(\underline{K})\underline{a}_|(\underline{k}) + c^2(\underline{K})\underline{a}_\perp(\underline{k}) \qquad (A.40)$$

The complete receiving vector $\underline{r}(\underline{K})$ is found from the requirement [A4, p.125]:

$$\underline{R}(\underline{K}) \cdot \underline{C}(\underline{K}) = \underline{r}(\underline{K}) \cdot \underline{c}(\underline{K}) \qquad (A.41)$$

(which states that z-components are redundant in describing the electromagnetic fields). Equation (A.37) gives:

$$\underline{r}(\underline{K}) = -[\gamma/k]R^1(\underline{K})\underline{a}_|(\underline{k}) + R^2(\underline{K})\underline{a}_\perp(\underline{k}) \qquad (A.42)$$

This can be checked by substituting (A.42) in (A.37):

$$\underline{r}(\underline{K}) \cdot \underline{c}(\underline{K}) = \{ -[\gamma/k]R^1(\underline{K})\underline{a}_|(\underline{k}) + R^2(\underline{K})\underline{a}_\perp(\underline{k}) \} \cdot$$
$$\cdot \{ -[k/\gamma]c^1(\underline{K})\underline{a}_|(\underline{k}) + c^2(\underline{K})\underline{a}_\perp(\underline{k}) \} =$$
$$= R^1(\underline{K})c^1(\underline{K}) + R^2(\underline{K})c^2(\underline{K}) =$$
$$= \{ R^1(\underline{K})\underline{K}_1 + R^2(\underline{K})\underline{K}_2 \} \cdot \{ c^1(\underline{K})\underline{K}_1 + c^2(\underline{K})\underline{K}_2 \} =$$
$$= \underline{R}(\underline{K}) \cdot \underline{C}(\underline{K}) \qquad (A.43)$$

Note that the following equation is valid too:

$$\underline{R}(\underline{K}) \cdot \underline{T}(\underline{K}) = \underline{r}(\underline{K}) \cdot \underline{t}(\underline{K}) \tag{A.44}$$

With use of (A.29), the desired relation between $\underline{r}(\underline{K})$ and $\underline{t}(\underline{K})$ will be derived.   .

According to (A.42):

$$\eta_0 k \underline{r}(\underline{K}) = -\eta_0 k [\gamma/k] R^1(\underline{K}) \underline{a}_|(\underline{k}) + \eta_0 k R^2(\underline{K}) \underline{a} \perp(\underline{k}) =$$

$$(A.29)$$
$$= +\eta_0 \gamma \eta_1 \eta_0^{-1} T^1(-\underline{K}) \underline{a}_|(\underline{k}) - \eta_0 k \eta_2 \eta_0^{-1} T^2(-\underline{K}) \underline{a} \perp(\underline{k}) =$$
$$= \gamma \eta_1 [\gamma/k][k/\gamma] T^1(-\underline{K}) \underline{a}_1(\underline{k}) - \eta_2 k T^2(-\underline{K}) \underline{a} \perp(\underline{k}) =$$
$$= \gamma[\omega\epsilon/\gamma][\gamma/k][k/\gamma] T^1(\underline{K}) \underline{a}_|(\underline{k}) - [\gamma/(\omega\mu)] k T^2(\underline{K}) \underline{a} \perp(\underline{k}) =$$
$$= [\omega\epsilon\gamma/k][k/\gamma] T^1(\underline{K}) \underline{a}_|(\underline{k}) - [\gamma k/(\omega\mu)] T^2(\underline{K}) \underline{a} \perp(\underline{k}) =$$
$$= \gamma\eta[k/\gamma] T^1(\underline{K}) \underline{a}_|(\underline{k}) - \gamma\eta T^2(\underline{K}) \underline{a} \perp(\underline{k}); \quad \eta = \sqrt{(\epsilon/\mu)} \quad (A.45a)$$

In the first step above, use is made of:

$$\eta_1 = \omega\epsilon/\gamma; \quad \eta_2 = \gamma/(\omega\mu) \tag{A.45b}$$

Now, first the unity vectors $\underline{a}_|(\underline{k})$ and $\underline{a} \perp(\underline{k})$ are considered.
With (22) and (5):

$$\underline{a} \perp(k) = \underline{K}_2 = \underline{a}_z \times \underline{K}_1 = \underline{a}_z \times \underline{K}/K$$
$$\Rightarrow \underline{a} \perp(-\underline{k}) = \underline{a}_z \times -\underline{K}/K = -\underline{a} \perp(\underline{k}) \tag{A.46a}$$

$$\underline{a}_|(\underline{k}) = \underline{K}_2 \times \underline{a}_k = \underline{a} \perp(\underline{k}) \times \underline{k}/k$$
$$\Rightarrow \underline{a}_|(-\underline{k}) = \underline{a} \perp(-\underline{k}) \times -\underline{k}/k =$$
$$= -\underline{a} \perp(\underline{k}) \times -\underline{k}/k = \underline{a}_|(\underline{k}) \tag{A.46b}$$

Equation (A.46) substituted in (A.45) yields:

$$\eta_0 k\underline{x}(\underline{K}) = \gamma\eta\{ \ [k/\gamma]T^1(-\underline{K})\underline{a}_|(-\underline{k}) + T^2(-\underline{K})\underline{a} \perp(-\underline{k}) \ \} -$$
$$- \gamma\eta\underline{t}(-\underline{K}); \quad \eta = \sqrt{(\epsilon/\mu)} \qquad\qquad (A.47)$$

which is the equation searched for.

A.3   References

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APPENDIX B   INTEGRAL EVALUATION [B1]

To be evaluated is the integral equation (37):

$$P_B(\underline{r}_0) = (\omega\mu)^{-1}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\{\underline{f}(\underline{K})\times(\underline{k}'\times\underline{g}(\underline{K}'))\} - \underline{g}(\underline{K}')\times\{\underline{k}\times\underline{f}(\underline{K})\}\}.$$

$$.(-\underline{a}_z)\exp(+j\underline{k}\cdot\underline{r} +j\underline{k}'\cdot\underline{r}')d\underline{K}d\underline{K}'dxdy' \qquad (B.1)$$

B.1   Evaluation

The vectors $\underline{k}$, $\underline{k}'$, $\underline{r}$ and $\underline{r}'$ can be written as:

$$\underline{k} = k_x\underline{a}_x + k_y\underline{a}_y + k_z\underline{a}_z \qquad\qquad\qquad (B.2a)$$

$$\underline{k}' = k_x'\underline{a}_x' + k_y'\underline{a}_y' + k_z'\underline{a}_z' \qquad\qquad\qquad (B.2b)$$

$$\underline{r} = x\underline{a}_x + y\underline{a}_y + z\underline{a}_z \qquad\qquad\qquad (B.2c)$$

$$\underline{r}' \quad x'\underline{a}_x' + y'\underline{a}_y' + z'\underline{a}_z' \qquad\qquad\qquad (B.2d)$$

The two rectangular coordinate systems are shown in figure B1 (see also figure 7).

$$\underline{a}_y \qquad \underline{a}'_y$$

$$\underline{a}_x \qquad \underline{a}_z \qquad \underline{a}'_z \qquad \underline{a}'_x$$
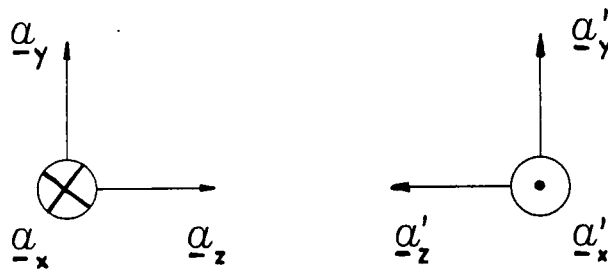
Fig. B1   Coordinate systems

Figure B1 shows that:

$$\underline{a}_x' = -\underline{a}_x \tag{B.3a}$$
$$\underline{a}_y' = \underline{a}_y \tag{B.3b}$$
$$\underline{a}_z' = -\underline{a}_z \tag{B.3c}$$

and, consequently:

$$\underline{k}' = -k_x'\underline{a}_x + k_y'\underline{a}_y - k_z'\underline{a}_z \tag{B.4a}$$
$$\underline{r}' = -x'\underline{a}_x + y'\underline{a}_y - b\underline{a}_z \tag{B.4b}$$

and:

$$\mathbf{r}_0 = \mathbf{r} \cdot \mathbf{r}' = (x+x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (a+b)\mathbf{a}_z =$$

$$= r_{0x}\mathbf{a}_x + r_{0y}\mathbf{a}_y + r_{0z}\mathbf{a}_z \qquad (B.5a)$$

$$\Leftrightarrow x' = r_{0x}-x; \ y' = y-r_{0y}; \ b = r_{0z}-a \qquad (B.5b)$$

The variables x and y appear only in the exponential term of the integrand. Integration with respect to them leads to:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(+j\mathbf{k}\cdot\mathbf{r} +j\mathbf{k}'\cdot\mathbf{r}')dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(+jk_x x+jk_y y+jk_z z+jk_x'x'+$$

$$+jk_y'y'+jk_z'z')dxdy =$$

$$= \exp(-j\mathbf{k}'\cdot\mathbf{r}_0 + ja(k_z-k_z'))\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(jx(k_x-k_x') + jy(k_y+k_y')dxdy$$

$$(B.6)$$

This is recognized as a double inverse Fourier integral. A single inverse Fourier integral looks like [B2,p.19]:

$$[1/(2\pi)]\int_{-\infty}^{\infty} \exp(+jyf)dy = \delta(f) \qquad (B.7a)$$

with $\delta(f)$ the Kronecker Delta function.

With (B.7a):

$$\int_{-\infty}^{\infty} \exp\{+jx(k_x-k_x')\}dx = 2\pi\delta(k_x-k_x') \qquad (B.7b)$$

Using this in equation (B.6) gives:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(j\mathbf{k}.\mathbf{r} + j\mathbf{k}'.\mathbf{r}')dxdy = (2\pi)^2\exp\{-j\mathbf{k}'.\mathbf{r}_0 + ja(k_z-k_z').$$

$$.\delta(k_x-k_x')\delta(k_y+k_y') \qquad (B.8)$$

and this result is substituted in (B.1) and integrated with respect to
k_x' and k_y'. In accordance with the sampling properties of the Dirac
function (δ), this leads to a simple evaluation of the rest of the
integral at $k_x' = k_x$; $k_y' = -k_y$. It will be noticed that the third
component of $\mathbf{k}'$ is:

$$k_z' = \sqrt{(k^2 - k_x'^2 - k_y'^2)} = \sqrt{(k^2 - k_x^2 - k_y^2)} = k_z \qquad (B.9)$$

and thus:

$$\mathbf{k}' = -\mathbf{k} \qquad (B.10)$$

Equation (B.1) can now be written as:

$$P_B(\mathbf{r}_0) = [4\pi^2/(\omega\mu)]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \{\mathbf{f}(\mathbf{K})\times\{\mathbf{k}\times\mathbf{g}(-\mathbf{K}) +$$

$$+ \mathbf{g}(-\mathbf{K})\times\{\mathbf{k}\times\mathbf{f}(\mathbf{K})\}\}.\mathbf{a}_z\exp(+j\mathbf{k}.\mathbf{r}_0)d\mathbf{K} \qquad (B.11)$$

in which use is made of:

$$\underline{k}' = k_x'\underline{a}_x' + k_y'\underline{a}_y' = k_x(-\underline{a}_x) + (-k_y)\underline{a}_y = -\underline{k} \qquad (B.12)$$

B.2   References

[B1] Van Hezewijk, J.G.:*Title Unknown*,
     To be published.

[B2] Brigham, E.O.: *The Fast Fourier Transform*,
     Prentice-Hall, Inc., 1974.

APPENDIX C METHOD OF STATIONARY PHASE

Consider the function I:

$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\exp(jkg(x,y))dxdy \qquad (C.1)$$

For a large k, the phase of I will vary rapidly and, under the condition that f(x,y) is a slowly varying function, I will tend toward zero when k approximates infinity.

If, however, there are points where g(x,y) is stationary, that is $\nabla g(x,y) = 0$, the term $\exp(jkg(x,y))$ will not vary rapidly in the vicinity of these points and the main contribution to I will be delivered in these points. By expanding the functions in I around these points of stationary phase, an approximation of I is found. According to [C1, pp.28-38] the approximation of I, I_{00} is:

$$I_{00} = [2\pi f/k][|Det(G_u)|]^{-1/2}\exp(j[kg+(\pi/4)(sgn(g_{u1u1}'') + $$
$$+ sgn(g_{u2u2}''))])|_{u1=0,\ u2=0} \qquad (C.2)$$

with:

$$Det(G_u) = g_{xx}''g_{yy}'' - g_{xy}''g_{xy}'' = $$
$$= \frac{\delta^2 g}{\delta x \delta x} \cdot \frac{\delta^2 g}{\delta y \delta y} - \frac{\delta^2 g}{\delta x \delta y} \cdot \frac{\delta^2 g}{\delta x \delta y} \qquad (C.3)$$

$\alpha \nu \delta e$

$$\begin{bmatrix} g_{u1u1}'' & 0 \\ 0 & g_{u2u2}'' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} g_{xx}'' & g_{xy}'' \\ g_{xy}'' & g_{yy}'' \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$(C.4a)$$

or:

$$g_{u1u1}'' = g_{xx}''\cos^2\theta + g_{yy}''\sin^2\theta - 2g_{xy}''\cos\theta\sin\theta \qquad (C.4b)$$

$$g_{u2u2}'' = g_{xx}''\sin^2\theta + g_{yy}''\cos^2\theta + 2g_{xy}''\cos\theta\sin\theta \qquad (C.4c)$$

$$0 = (g_{xx}''-g_{yy}'')\sin\theta\cos\theta + g_{xy}''(\cos^2\theta - \sin^2\theta) \qquad (C.4d)$$

Equation (C.4d) yields, see also [C1, pp.28-38]:

$$\theta = (1/2)\tan^{-1}[2g_{xy}''/(g_{yy}'' - g_{xx}'')] \qquad (C.5)$$

To apply the theory above, equation (52) is rewritten:

$$E(r) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} f(k_x,k_y)\exp(jrg(k_x,k_y))dk_x dk_y \qquad (C.6a)$$

with:

$$\mathbf{f}(k_x,k_y) = \mathbf{t}(\mathbf{K}) \tag{C.6b}$$

$$g(k_x,k_y) = k_x x/r + k_y y/r + k_z z/r \tag{C.6c}$$

The derivatives with respect to k_x and k_y are found to be (with $k_z = \sqrt{(k^2 - k_x^2 - k_y^2)}$):

$$g_{kxkx}{}'' = -[z/r][(k_z^2 + k_x^2)/k_z^3] \tag{C.7a}$$

$$g_{kyky}{}'' = -[z/r][(k_z^2 + k_y^2)/k_z^3] \tag{C.7b}$$

$$g_{kxky}{}'' = -[z/r][k_x k_y/k_z^3] \tag{C.7c}$$

So:

$$Det(G_u) = g_{kxkx}{}'' g_{kyky}{}'' - g_{kxky}{}'' g_{kxky}{}'' = \\ = [z^2/r^2][k^2/k_z^4] \tag{C.8}$$

$$\theta = (1/2)\tan^{-1}[2k_x k_y/(k_y^2 - k_x^2)] \tag{C.9}$$

The stationary phase is found for $\nabla g=0$. This implicates:

$$\nabla g(k_x,k_y) = \underline{0} \; \Rightarrow \; \mathbf{k} = \mathbf{k}_0 = k\mathbf{a}_r \tag{C.10a}$$

This is true because:

$$\nabla g(k_x, k_y) = \frac{\delta g}{\delta k_x} \underline{a}_x + \frac{\delta g}{\delta k_y} \underline{a}_y =$$

$$= [\frac{x}{r} - \frac{k_x}{k_z} \frac{z}{r}] \underline{a}_x + [\frac{y}{r} - \frac{k_y}{k_z} \frac{z}{r}] \underline{a}_y =$$

$$= \underline{0} \qquad (C.10b)$$

or:

$$k_x = k_z \frac{x}{z}; \quad k_y = k_z \frac{y}{z} \qquad (C.10c)$$

k_z follows from:

$$k_z^2 = k^2 - k_x^2 - k_y^2 = k^2 - k_z^2 (\frac{x^2 + y^2}{z^2}) \qquad (C.10d)$$

or:

$$k_z = k/\{z^2/(x^2+y^2+z^2)\} = k[z/r] \qquad (C.10e)$$

Substitution in (C.10c) yields:

$$k_x = k[x/r]; \quad k_y = k[y/r] \qquad (C.10f)$$

so: $\underline{k}$ and $\underline{r}$ have the same direction. This means for (C.8):

$$\text{Det}(G_u) - [z^2/r^2][k^2/k_z^4] - [1/k_{z0}^2][k_0^2/k_{z0}^2][z^2/r^2] -$$
$$- [1/k_{z0}^2][(k_{x0}^2+k_{y0}^2+k_{z0}^2)/k_{z0}^2][z^2/(x^2+y^2+z^2)] -$$
$$- [1/k_{z0}^2][(k_{x0}^2+k_{y0}^2+k_{z0}^2)/k_{z0}^2][k_{z0}^2/(k_{x0}^2+k_{y0}^2+k_{z0}^2)] -$$
$$- 1/k_{z0}^2 \qquad\qquad (C.11)$$

Equation (C.7) substituted in (C.4) with x and y replaced by k_x and k_y, respectively, gives:

$$g_{u1u1}{}'' - -[z/(rk_z^3)][k_z^2 + (k_x\cos\theta - k_y\sin\theta)^2] \qquad (C.12a)$$
$$g_{u2u2}{}'' - -[z/(rk_z^3)][k_z^2 + (k_x\cos\theta + k_y\sin\theta)^2] \qquad (C.12b)$$

Because $r - \sqrt{(x^2+y^2+z^2)} > 0$ and $\text{sgn}(z) - \text{sgn}(k_z) - \text{sgn}(k_z^3)$, for $k_z \in R$ (the latter following from (C.10)):

$$\text{sgn}(g_{u1u1}{}'') - \text{sgn}(g_{u2u2}{}'') - -1 \qquad\qquad (C.13)$$

and so (C.6) becomes:

$$\underset{r \to \infty}{\underline{E}(\underline{r})} \approx [2\pi/r]\underline{t}(\underline{K})[|1/k_{z0}^2|]^{-1/2}\exp\{j[r(k_xx/r + k_yy/r + k_zz/r) +$$
$$+ (\pi/4)(-1-1)]\} -$$

$$- -j2\pi k\cos\theta\,\underline{t}(\underline{K})\frac{e^{+jkr}}{r} \qquad\qquad (C.14)$$

in which use is made of:

$$k_{x0}x + k_{y0}y + k_{z0}z - \underline{k}_0\cdot\underline{r} - k\underline{a}_r\cdot r\underline{a}_r - kr \qquad\qquad (C.15)$$

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THE THEORY OF PLANAR NEAR-FIELD MEASUREMENT IS DISCUSSED FOR TWO DIFFERENT APPROACHES: THE SCATTERING-MATRIX FORMULATION APPROACH AND THE RECIPROCITY THEOREM APPROACH. THE INTERACTION OF TEST ANTENNA AND PROBE ANTENNA IS CALCULATED, IN WHICH IS TAKEN ACCOUNT OF THE PRESENCE OF THE PROBE USED TO SAMPLE THE FIELD DISTRIBUTIONS SINCE THE PROBE IS NOT ASSUMED TO BE IDEAL (ELEMENTARY MAGNETIC OR ELECTRIC DIPOLE), IT IS SAID THAT PROBE CORRECTION IS APPLIED.

IT IS SHOWN THAT BOTH APPROACHES LEAD TO EQUAL RESULTS. THE DERIVATION OF FAR-FIELD ANTENNA CHARACTERISTICS FROM NEAR-FIELD DATA IS THEN DISCUSSED AND NEAR-FUTURE RESEARCH ITEMS ARE SHORTLY MENTIONED.

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